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| **Document Title:** WORKING DOCUMENT PRELIMINARY DRAFT REVISION OF RECOMMENDATION ITU-R M.1851-1  **Mathematical models for radiodetermination radar and aeronautical mobile systems antenna patterns for use in interference analyses** | |
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| **Purpose/Objective:** Update the Cosecant squared pattern description that is currently being used in the studies of AI 1.4 with FAA radar systems.  Also, add peak and average parabolic antenna sidelobe patterns equations. | |
| **Abstract:** This work is initiated because it was noticed in WP-5D AI 1.4 HIBS studies the cosecant squared pattern was not clearly defined in M.1851. So, this needs to be fixed.  We will also add the peak and average patterns for a circular parabolic dish antenna with several Bessel function antenna tapers. This work will be similar form to the linear aperture cases that is included in M.1851.  These Parabolic antenna patterns are expected to be also useful for interference and compatibility studies for radar, AMS and other systems that use this type of antennas. | |
| **Fact Sheet Preparer:** Raafat Nasser, ACES Inc. for FAA | |

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Mathematical models for radiodetermination radar and aeronautical mobile systems antenna patterns for use in interference analyses

Scope

The revision of this Recommendation updates the Cosecant-Squared antenna pattern and adds parabolic antenna distribution mathematical models for radiodetermination radar and aeronautical mobile system antenna patterns to be used for single-entry and aggregate interference analysis. Depending on the antenna 3 dB beamwidth and first peak side-lobe level, the proper set of equations for both azimuth and elevation patterns may be selected for the purpose of interference analysis. Both Peak sidelobe envelope patterns for single entry interferer and average sidelobe envelope patterns for multiple interferers are defined.

**Proposal**

The United States proposes to update the cosecant-squared equations description and add the parabolic antenna patterns equations to the ITU-R M.1851-1.

Mathematical models for radiodetermination radar and aeronautical mobile systems antenna patterns for use in interference analyses

[Editor’s Note: No other changes are required to the ITU-R M.1851 prior to this section. Author requests to replace section 2.2 with the following section to its entirety]

## 2.2 Antenna with a cosecant-squared elevation pattern

The cosecant-squared pattern is a special case. The power (not field-strength) is given by:

(2.2-1)

where:

*G*(θ): cosecant squared pattern between angles of and

: pattern gain at

: elevation (or depression) of the half-power point on the main lobe where cosecant-squared pattern starts for ground radar and for airborne radar. If the radar operational requirements are provided, then use as shown below

: the gain pattern value at is given by . When is the value of , then the gain value is 0.5 or -3 dB

: one-half antenna Null-to-Null beamwidth given by in degrees. Using the antenna beam pointing angle, the value for is in degrees for ground radar and for airborne radar. This defines the lowest value of the pattern

: maximum angle where cosecant-squared pattern stops

θ: angle to evaluate the antenna pattern (degrees)

: half power antenna beamwidth (degrees)

: antenna beam tilt elevation angle or beam pointing angle (degrees).

If the operational maximum range and height values for a radar system application are provided, then the angle where the CSC2 starts is given by (see chapter 2 of Radar Equations for Modern Radar by David K. Barton 2013 Artech House):

(2.2-2)

Where is the start angle of the cosecant squared pattern replacing and is the Earth radius of 6378 km. However, if the operational parameters are not provided then the CSC2 start angle is given by the following equations:

for ground radar and for airborne radar.

The cosecant pattern is applied as shown in Table 2.2-1:

TABLE 2.2-1

Cosecant-squared antenna pattern equations for Ground and Airborne Radars

| Cosecant-squared equation | Condition Airborne Radar Ground Radar | Comment | Equation No. |
| --- | --- | --- | --- |
| Cosecant floor level (example = −55 dB + Peak antenna gain) | Airborne Radar  Ground Radar | At angles less than use -55 dB front to back ratio (-55 dB+Peak antenna gain) Note that is in degrees for ground radar and for airborne radar |  |
| ; | Airborne Radar  Ground Radar | Use from the lower one half the null to-null beamwidth to the start of the CSC2 pattern at whichever is provided Note: for ground radar and for airborne radar. | (2.2-3) |
|  | Airborne Radar | Start the CSC2 pattern up to the maximum CSC2 angle | (2.2-3) |
| Cosecant floor level (example = −55 dB + Peak antenna gain) | Airborne Radar | At angles greater than use -55 dB front to back ratio (-55 dB+Peak antenna gain) | (2.2-4) |
|  | Antenna gain where the CSC2 patterns starts | The gain at is the gain of the pattern at . The pattern gain is lower than the peak antenna gain by 3 dB at  Note for ground radar and for airborne radar. | (2.2-5) |

*Note that* refers to the amplitude of the power pattern, while  and refer to the ‘Directivity pattern F(μ)’, field amplitude; which are square of power amplitude.

The solution might be writing and 

A graphical description of the patterns is shown in the Figures below.

Figure 2.2-1

Cosecant squared beam coverage for ground search radar

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Figure 2.2-2

Cosecant squared beam coverage for Airborne radar

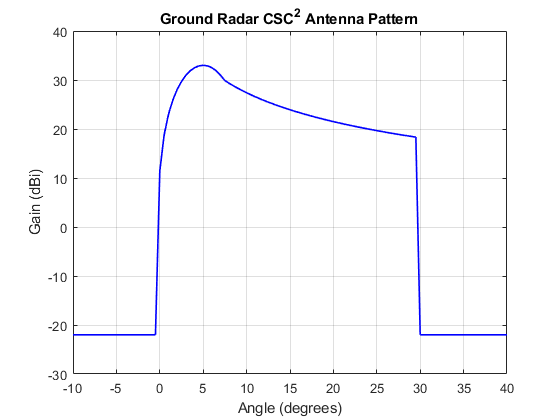
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An example using the above procedure provides an antenna pattern for radar C and is shown below.

Figure 2.2-3

Radar-C Cosecant Squared (CSC2) Beam Pattern centred at +5°



For airborne radar the inverted CSC2 antenna pattern is shown below.

Figure 2.2-4

Example Airborne Cosecant Squared (CSC2) Beam Pattern centred at -10°

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[Editor’s Note: No additional changes up to section 4.1. Section 4.1 is all new]

# **4.1 Parabolic Taper Aperture Antenna**

This section describes parabolic taper aperture antenna peak and average envelope radiation patterns for use in interference analyses. When information on the antenna half-power beamwidth and peak side-lobe level are provided, the proper set of equations for peak and average patterns may be selected. Peak sidelobe envelope patterns are used for single entry interferer and average sidelobe envelope patterns is used for multiple interferers.

**4.1.1 Parabolic Antenna use and Pattern description**

A parabolic antenna is an antenna that uses a parabolic reflector, a curved surface with the cross-sectional shape of a parabola, to direct the radio waves. The most common reflector antennas are the corner reflector antenna, parabolic antenna, paraboloidal antenna, and Cassegrain antenna. Parabolic antennas can have some of the highest gains and narrowest beamwidths of any antenna type. To achieve narrow beamwidths, the parabolic reflector diameter must be much larger than the wavelength of the radio waves used.

Parabolic antennas are used for point-to-point communications, wireless links for data communications, satellite communications and spacecraft communication antennas. They are also used in radio telescopes. The ITU-R has many recommendations for these applications.

The focus here is on the use of parabolic antennas in radar applications, in which there is a need to transmit a narrow beam of radio waves to locate objects or to communicate with ships or airplanes for example. For ITU-R compatibility and interference studies there is a need to develop a peak and average antenna sidelobe pattern envelops that only depend on the known antenna half-power beamwidth and peak sidelobe level. The approach used is similar to that of the linear aperture.

From references 1 and 3, the normalized pattern function for parabolic distribution for different tapers is given by the following equation.

(4.1.1-1)

where

is the free space constant =

a is the radius of the antenna.

D is the antenna diameter

is the antenna pattern half power beamwidth.

In equation 17, becomes

is the Bessel function (can be evaluated in Excel or Matlab etc.).

in the wavelength, and

n is the parabolic taper power value.

The antenna dimension can be eliminated using , where the values for K in degrees are provided in Table 4.1.1-1 where . Equation 4.1.1-1 can be written as

(4.1.1-2)

For the parabolic distribution, we have the following relationships as provided in the table below from reference 1 where the normalized pattern is a function of the beamwidth and the estimated peak sidelobe level.

TABLE 4.1.1-1

**Radiation Pattern Characteristics Produced by Circular Aperture Distributions (Reference-1)**

|  |  |  |
| --- | --- | --- |
| **Parabolic Power, n** | **Peak Sidelobe Level (dB)** | **K (°)** |
| 0 | -17.66 | 58.2125 |
| 1 | -24.64 | 72.5938 |
| 2 | -30.61 | 84.0529 |
| 3 | -35.96 | 96.3142 |
| 4 | -40.0 | 108.2317 |

### **4.1.2 Procedure to compute sidelobe envelope**

Using equation 4.1.1-2 and Table-4.1.1-17, it is possible to develop the mask equations. These masks are derived using curve fits to the antenna peak side‑lobe levels beyond the antenna pattern first null location. It has been found, by comparing the integral of the theoretical and the proposed mask patterns, that the difference between the peak and average envelopes in one principal plane cut is 6 dB. The following procedure is used for calculating the peak and average envelops:

1. compute equation 4.1.1-2 for different n values using the value of K from Table 4.1.1-1 then normalize the pattern and convert to dB using ;
2. to plot the mask, use the theoretical directivity pattern from equation 4.1.1-2 up to the break point for either the peak or average antenna pattern, as required. After the break point, apply the mask pattern as indicated in Table 4.1.2-1;
3. the peak pattern mask is the antenna pattern that rides over the side-lobe peaks. It is used for a single-entry interferer;
4. the average pattern mask is the antenna pattern that approximates the integral value of the theoretical pattern. It is used for aggregated interferers;
5. the average pattern mask break point is the point in pattern magnitude (dB) below the maximum gain where the pattern shape departs from the theoretical pattern into the average mask pattern;
6. the peak pattern mask break point is the point in pattern magnitude (dB) below the maximum gain where the pattern shape departs from the theoretical pattern into the peak mask pattern;
7. θ3 is the 3 dB antenna beamwidth (degrees);
8. θ is the angle in either the elevation (vertical) or azimuth (horizontal) principal plane cuts (degrees); and
9. the average mask is computed using the peak mask and subtracting approximately 6 dB. Note that the break points of the peak pattern are different from the average patterns.

Table 4.1.2-1 shows the equations to be used in the calculations of the average and peak antenna masks.

TABLE 4.1.2-1

Peak and average theoretical mask pattern equations

| Field distribution Equation-4.1.1-2 | Mask equation beyond pattern break point where mask departs from theoretical pattern (dB) | Peak pattern break point where mask departs from theoretical pattern (dB) | Average pattern break point where mask departs from theoretical pattern (dB) | Constant added to the peak pattern to convert it to average mask (dB) | Mask Front-to-Back floor level (dB) | Equation No. |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | −6 | −35 | (4.1.1-3) |
|  |  |  |  | −6 | -50 | (4.1.1-4) |
|  |  |  |  | −6 | −60 | (4.1.1-5) |
|  |  |  |  | −6 | −70 | (4.1.1-6) |
|  |  |  |  | −6 | −80 | (4.1.1-7) |

The approach is shown in figure-4.1.2-1.

FIGURE 4.1.2-1

Break Point Example

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The following graphs show the results for field distribution of n=0 to n=4.

FIGURE 4.1.2-2

Example for n=0

Chart

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FIGURE 4.1.2-3

Example for n=1

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FIGURE 4.1.2-4

Example for n=2

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FIGURE 4.1.2-5

Example for n=3

Chart, line chart

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FIGURE 4.1.2-6

Example for n=4

Chart, line chart

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### **4.1.3 Antenna Pattern Selection**

Table 4.1.3-1 provides suggestion for how the antenna pattern should be selected based on information about half‑power beamwidth and peak side-lobe level.

TABLE 4.1.3-1

Parabolic Taper as a function of Peak Sidelobe Level

| Range of normalized peak side-lobe level (dB) | Parabolic Antenna Distribution power n | Equation No. |
| --- | --- | --- |
| -35 to -45 | n = 0 | (4.1.1-3) |
| -45 to -55 | n = 1 | (4.1.1-4) |
| -55 to -65 | n = 2 | (4.1.1-5) |
| -65 to -75 | n = 3 | (4.1.1-6) |
| Less than -75 | n = 4 | (4.1.1-7) |

**References**

1. Handbook of Radar Measurement by David K. Barton and Harold R. Ward Artech House 1984 Table A.15 page 264.
2. Wiley encyclopaedia of RF and microwave engineering, 6 Volume Set / Kai Chang, editor-in-chief section on Aperture Antennas” by Dennis Koazkoff, Devry University Alpharetta, Georgia page 365. The Encyclopaedia of RF and Microwave Engineering is available online at http://www.mrw.interscience.wiley.com/erfme.
3. Antenna theory and design by Warren L. Stutzman, Gary A. Thiele. — 3rd ed. 2013. Table 9-2 Page 389.

[Editor’s Note: No other changes beyond this part].